Wavelength conversion bandwidth enhancement through quasi-phase-matching in a width modulated silicon waveguide

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Abstract: Width-modulated silicon waveguide is investigated as a means of quasi-phase-matching to enhance the bandwidth of wavelength conversion based on four-wave-mixing. A conversion bandwidth enhancement of ~40% is achievable in a 5mm sinusoidally modulated SOI waveguide.

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1. Introduction

Four-wave-mixing (FWM) in silicon waveguide has been intensively investigated as a solution to wavelength conversion [1-3]. In this process, the signal is converted to the idler at a different wavelength with the aid of the degenerate or non-degenerate pump [4]. In most cases, the signal is placed in adjacent to the pump, which results in a symmetric flat conversion band in the center lobe [5]. However, discrete band frequency conversion is also explored to facilitate conversion in the sidebands, especially for applications involving great signal-idler spacing, e.g., conversion between near-infrared and mid-infrared [6-8]. The FWM is a parametric process sensitive to the accumulative phase difference between the co-propagating waves. The wavelength conversion bandwidth is hence intrinsically limited by the dispersion [4]. However, it is still possible to design a waveguide with flat dispersion slope near the zero-dispersion wavelength (ZDWL) in favor of wavelength conversion band [3,5]. Meanwhile, the proper spacing of the non-degenerate pumps can also broaden the bandwidth with respect to FWM with degenerate pump [9]. Recently, quasi-phase-matching (QFM) is theoretically and experimentally demonstrated in silicon waveguides, including using anisotropic silicon nonlinearity induced Kerr grating in ring resonator or cascaded waveguide bends [10,11], and binarily or sinusoidally varying width to modulate the waveguide dispersion [8,12]. However, all the research has been focused on QFM’s capability on boosting the conversion efficiency. No work has been reported on utilizing QFM to enhance the bandwidth, which is another important metrics in any wavelength conversion process. In this report, we investigate the possibility of using QFM in silicon waveguide to enhance the wavelength conversion bandwidth (CBW). The waveguide is periodically modulated in width to produce group-velocity-dispersion continually varying between positive and negative along the waveguide. In this way, the phase difference between pump, signal and idler is compensated periodically, resulting in a continuous energy transfer from the pump to the signal and idler. This phase is completely or partially compensated in a wide bandwidth near the ZDWL and the CBW is therefore broadened. Our simulation shows that the CBW is enhanced by ~40% by QFM with respect to the highest achievable CBW in a uniform waveguide with the same height of 300nm.

2. Width modulation and bandwidth enhancement

Wavelength conversion based on FWM is determined by the phase-matching condition expressed as \( \Delta k = \Delta \beta + \Delta k_{\text{non-linear}} = (\beta_s + \beta_i - 2 \beta_p) + 2 \gamma P_p \), where \( \beta \) is the propagation constant and the subscript \( s, i \) and \( p \) stand for signal, idler and pump respectively, \( \gamma \) is the effective nonlinearity and \( P_p \) the pump power. \( \Delta k \) is dominated by the linear phase mismatch, which is directly evaluated from \( \Delta \beta = \frac{2 mn_{\text{eff},s}}{\lambda_s} + \frac{2 mn_{\text{eff},i}}{\lambda_i} - 2 \cdot \frac{mn_{\text{eff},p}}{\lambda_p} \), in which \( \lambda \) is the wavelength and \( n_{\text{eff}} \) is the effective refractive index. \( n_{\text{eff}} \) is calculated using a Finite-element Method incorporating the Sellmeier material dispersion equation for both core and substrate. As the wavelength spacing between pump and signal/idler increases, the phase matching condition begins to increasingly deviate from \( \Delta k = 0 \) [4]. As shown in Fig. 1(a), waveguides with different width demonstrate different \( \Delta \beta \) tendency and they can be cascaded to compensate between positive and negative \( \Delta k \)'s to possibly achieve QFM across a wider band. To evaluate the bandwidth enhancement by QFM, the wavelength conversion is characterized by numerically solving the Schrodinger Equation using the fourth order Runge-Kutta method. The linear loss, two-photon absorption (TPA), TPA induced free-carrier absorption (FCA), and self-phase and cross-phase modulations are taken into account [4,20]. Since the waveguide width is varying along the waveguide to facilitate QFM, the linear phase mismatch \( \Delta \beta(z) \) is a function of \( z \) and the phase accumulation is the integration over the waveguide length. As an example
shown in Fig. 1(b1), the waveguide width is sinusoidally varied between 307nm and 325nm with $W_{DC} = 316\text{nm}$ and $\Delta W = 18\text{nm}$ according to

$$W(z) = W_{DC} + \frac{\Delta W}{2} \sin \left( \frac{2\pi z}{L} + \frac{\pi}{2} \right)$$  \hspace{1cm} (1)$$

$W_{DC}$ is the average width; $\Delta W$ is the width swing; $L$ is the waveguide length and it’s divided into $N$ periods of sine sections. Fig. 1(b2) shows the corresponding $\Delta \beta(z)$ with signal wavelength of 1.5$\mu$m and pump wavelength of 1.55$\mu$m. The linear phase mismatch alternates between positive and negative and results in an accumulative linear FWM phase difference of $\Delta \Phi_{FWM} = \int_0^L \Delta \beta(z) dz$ as shown in Fig. 2(b3). The phase difference between signal, idler and pump is periodically compensated so that $\Delta \Phi_{FWM}$ is constrained within $\left( -\frac{\pi}{2}, \frac{\pi}{2} \right)$. The energy is therefore continuously transferred from the pump to the idler and the idler power increases monotonically as a result of QFM (Fig. 1(b4)). Without QFM, the phase difference will enter the $\left( \frac{\pi}{2}, \frac{3\pi}{2} \right)$ region, in which the idler/signal power will be reversely transferred back to the pump. In this case, the phase mismatch is fully compensated to demonstrate the concept of QFM. In most other cases, it can be partially compensated, but still results in a slower accumulation of phase difference.

![Fig. 1(a) Linear phase mismatch $\Delta \beta$ for different signal wavelength in strip waveguides with different width ($\lambda_p = 1.55\mu m$); (b1) The width profile of a typical width-modulated strip waveguide ($W_{DC} = 316\text{nm}$ and $\Delta W = 18\text{nm}$); (b2) the corresponding linear phase mismatch; (b3) periodic compensation of the linear FWM phase difference; (b4) the idler power increases monotonically as a result of QFM.](image)

However, the width modulation doesn’t necessarily guarantee bandwidth enhancement. The modulation parameters, $W_{DC}$, $\Delta W$ and $\Lambda$ in this sinusoidal modulation case, need to be properly chosen to ensure the QFM condition over a wide bandwidth. To investigate the effects of these parameters, the CBW is calculated for a combination of different $W_{DC}$, $\Delta W$ and $\Lambda$. Fig. 2(a) shows the CBW dependence on $W_{DC}$ for different width swings. $\Delta W = 0$ corresponds to a uniform straight waveguide and the maximum CBW of 104nm in this conventional strip waveguide is achieved at $W_{DC} = 314\text{nm}$. As the $\Delta W$ increases in width-modulated waveguides, the corresponding CBW peak maps to larger waveguide width offset $W_{DC}$, e.g. 314.5nm, 316nm and 318nm for $\Delta W = 4\text{nm}$, 16nm and 28nm respectively. This is because $\Delta \beta$ is more sensitive to $W$ when $W$ is small than the case with larger $W$ values as shown in Fig. 1a and Fig. 1b2, and the optimal $W_{DC}$ has to move to greater values to ensure better phase compensation. More interestingly, increasing $\Delta W$ does not necessarily increase the CBW. Although this is the case when width swing is small, the maximum CBW of 124nm for $\Delta W = 28\text{nm}$ gets smaller than 134nm for $\Delta W = 16\text{nm}$. This is caused by the nonlinear nature of the dependence of $\Delta \beta$ on waveguide width.

Fig. 2(b) shows the dependence of CBW on the number of periods along the waveguide length. A CBW of 140nm can be achieved, meaning an enhancement of $\sim 40\%$ with respect to the $\sim 100\text{nm}$ CBW achievable in the conventional uniform strip waveguides. Generally, higher number of periods gives rise to larger CBW’s. This tendency is especially pronounced when there are only a few periods. For a waveguide with fix length, smaller number of periods means a longer period and it becomes more stringent to pin the accumulated phase difference within $\left( -\frac{\pi}{2}, \frac{\pi}{2} \right)$ for longer periods. When the period is small enough, the phase difference accumulated in the positive or negative $\Delta \beta$ region from each half period will not exceed the $\frac{\pi}{2}$ limit and the QFM condition will be
determined by the net phase mismatch in the whole period. Therefore, the CBW saturates at large number of periods and there is little room for further bandwidth enhancement by introducing more than 10 periods. However, it is still worthy of note that greater waveguide width swing possess higher potential for CBW enhancement with large number of periods. For example, $\Delta W = 28 \text{nm}$ could provide higher CBW than $\Delta W = 16 \text{nm}$ with 20 sinusoidal periods.

![Figure 2](image)

**(a)** The dependence of conversion bandwidth on the waveguide width offset $W_{DC}$ for different width swings. The waveguide is 0.5mm long with 10 periodic sections. (b) The conversion bandwidth increases with higher number of periods and saturates when it is greater than ~10.

### 3. Summary

Width-modulated silicon waveguide is investigated as a means of quasi-phase-matching to enhance the bandwidth of wavelength conversion based on four-wave-mixing. Sinusoidal modulation is studied for its simplicity and the mechanism can still extends to more complicated modulation schema, e.g. tapered sine and sinusoidal Fourier series, to achieve possibly better result. Our simulation shows a conversion bandwidth enhancement of ~40% is achievable using this scheme of quasi-phase-matching. The maximum achievable CBW corresponds to a width offset $W_{DC}$ increasing with greater modulation depth $\Delta W$. It is also shown that CBW improves as we increase the number of periods contained in the waveguide length. However, it will saturate at large period number and there is little benefit introducing more than 10 periods for a waveguide with a length of 5mm.

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### 5. References


